# The package for TeX and LATEX

# tuple

v0.2

2024/12/20

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This extension provides common operations for tuples of numbers, in a expandable way, with a concise and easy-to-use "object.method" syntax.

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## 1 Preview

We consider the list of numbers (which we will now call a "tuple"):

```
12.7, 6.3, 11.7, 2.9, 5.5, 8.1, 4.3, 9.4, 13.6, 2.9, 6.9, 11.2, 5.1, 7.7, 10.1, 3, 4.1
```

We can define an "object", which we name for example "nn" with the instruction:

```
\newtuple{nn}{12.7, 6.3, 11.7, 2.9, 5.5, 8.1, 4.3, 9.4,13.6, 2.9, 6.9, 11.2, 5.1, 7.7, 10.1, 3, 4.1}
```

This tuple "nn" will be used throughout this documentation and recalled in comments in all the codes where it occurs. Here is its maximal value:

```
1) \tplexe{nn.max} \par 1) 13.6
2) \edef\foo{\tplexe{nn.max}}\meaning\foo 2) macro:->13.6
```

We can also calculate the median of the 5 smallest values, which supposes:

- 1. to sort the tuple (method sorted);
- 2. to retain only the values whith index 0 to 4 (method filter);
- 3. to find the median of the 5 numbers retained (method med).

Without LATEX, this package is loaded with

```
\input tuple.tex
```

and under LATEX with

\usepackage{tuple}

This package does not rely on any other except the LaTeX3 module "l3fp", which is now part of the LaTeX kernel, in particular to take advantage of its powerful macro \fpeval.

If we do not use LATEX, tuple will load the file expl3-generic.tex in order to have the l3fp module.

# 2 Declaring a tuple object

The macro \newtuple{ $\langle name \rangle$ }{ $\langle list\ of\ numbers \rangle$ } allows to construct a tuple "object" that is then accessed by its  $\langle name \rangle$ .

The  $\langle name \rangle$  accepts all alphanumeric characters [az][AZ][09], spaces and punctuation. Spaces preceding or following it are removed. A macro is also allowed<sup>2</sup>.

The  $\langle list\ of\ numbers \rangle$  is fully expanded and then detokenized before being used by the constructor of the tuple object. Empty elements are ignored.

For the sake of simplicity or speed<sup>3</sup>, the user is given the choice of specifying the type of numbers that make up the tuple using the macro \tplsetmode;

- \tplsetmode{int} to request that all numbers be signed integers between  $-2^{31} + 1$  and  $2^{31} 1$ . Comparisons during sorting are made with \ifnum, operations on numbers with \numexpr.
- \tplsetmode{dec short} specifies "short" decimals, i.e. 8-digit intpart and 8-digit decpart. In this case, comparisons are made with \ifdim and operations on numbers by a rudimentary calculation engine embedded within tuple.
- \tplsetmode{dec long}, which is the default mode, specifies "long" decimals in the sense of l3fp. In this mode, comparisons and operations on numbers are performed by l3fp.

Whichever mode is selected, final calculations (standard deviation, mean, quartile, etc.) are performed by l3fp.

 $<sup>^2</sup>$ The macro will never be modified by the package tuple: internally, this macro is detokenized to build a more complex name of a macro.

<sup>&</sup>lt;sup>3</sup>The speed differences aren't huge, but they do exist, see page 8

It is possible to define an empty tuple (i.e. one that does not contain any numbers), but many methods will return an error if they are executed on an empty list.

On the other hand, it's impossible to redefine an existing tuple (this requires the store method).

## 3 Methods

Here is the syntaxe to execute methods on a tuple:

```
\t \{ \langle tuple \ name \rangle . \langle method \ 1 \rangle . \langle method \ 2 \rangle . . . \langle method \ n \rangle \}
```

No spaces are allowed between the "." and the name of a method. It is therefore illegal to write ".∟sorted".

There are 3 types of datas for the tuple package:

- 1. numbers (and displayable datas);
- 2. "tuple" objects;
- 3. the "storage" type which characterizes non-expandable methods performing assignments.

All methods in this package take a tuple as input (which is the result of the previous methods) and return a result whose type determines which group the method belongs to:

- group 1 "tuple → number";
- group 2 "tuple → tuple". The methods in this group *do not modify* the initial tuple<sup>4</sup>, they act on a temporary tuple that it is obviously possible to save with a method in the group below;
- group 3 "tuple → storage";

The macro \tplexe and its argument are expandable, provided that the methods invoked are not in the group "tuple → storage".

If no method is specified, an expandable, implicit and generic method of the group "tuple  $\longrightarrow$  number", is executed and returns the tuple.

## 4 Methods of the group tuple → number

#### 4.1 Methods len, sum, min, max, mean, med and stdev

All these expandable methods do not accept any argument and return respectively the number of elements, their sum, the minimum, the maximum, the mean, the median and the standard deviation.

```
%\newtuple{nn}{12.7, 6.3, 11.7, 2.9, 5.5, 8.1, 4.3, 9.4,
                13.6, 2.9, 6.9, 11.2, 5.1, 7.7, 10.1, 3, 4.1}%
                                                                                  1) len = 17
1) len =
            \tplexe{nn.len}\par
                                                                                  2) sum = 125.5
2) sum =
            \tplexe{nn.sum}\par
                                                                                  3) min = 2.9
3) min =
            \tplexe{nn.min}\par
                                                                                  4) max = 13.6
            \tplexe{nn.max}\par
4) max =
                                                                                  5) mean = 7.382352941176471
5) mean =
            \tplexe{nn.mean}\par
                                                                                  5) \text{ med} = 6.9
5) \text{ med} =
            \tplexe{nn.med}\par
                                                                                  6) stdev = 3.447460325944583
6) stdev = \tplexe{nn.stdev}
```

## 4.2 Method quantile

This expandable method has the syntax

```
quantile\{\langle p \rangle\}
```

<sup>&</sup>lt;sup>4</sup>They cannot do so, otherwise they would not be expandable!

where  $\langle p \rangle$  must be a number between 0 and 1. The method returns the quantile according to the argument  $\langle p \rangle$ . The method used is the average method<sup>5</sup>; this is interpolation scheme "R7" described in this article.

```
%\newtuple{nn}{12.7, 6.3, 11.7, 2.9, 5.5, 8.1, 4.3, 9.4,
% 13.6, 2.9, 6.9, 11.2, 5.1, 7.7, 10.1, 3, 4.1}% 1) 4.3
1) \tplexe{nn.quantile{0.25}}\par 2) 6.9
2) \tplexe{nn.quantile{0.5}}\par 2) 10.1
2) \tplexe{nn.quantile{0.75}}\par
```

Note that quantile {0.5} is equivalent to med.

It's important to note that spaces before and after method arguments are ignored. It is therefore possible to write  $.quantile_{\sqcup}\{0.5\}_{\sqcup}$ .

## 4.3 Method get

This expandable method has the syntax

```
get{\langle index \rangle}
```

The first index is 0 and the last is n-1 where n is the number of elements in the tuple. Therefore, the argument of get must be between 0 and n-1. You can also use negative indexes, where -1 is the index of the last element, -2 that of the penultimate element and so on up to -n.

The argument of get is evaluated before being used: it is therefore possible to put the expandable macro \tplexe with a final method returning an integer.

## 4.4 Method pos

This expandable method has the syntax

```
pos{\langle number \rangle}[\langle n \rangle]
```

and returns the index of the  $\langle n \rangle$  occurrence of the  $\langle number \rangle$  in the tuple. If the tuple does not contain the  $\langle number \rangle$ , -1 is returned. If the optional argument is not present,  $\langle n \rangle$  is -1.

#### 4.5 Method show

This expandable method does not accept any arguments and is intended to convert a tuple object into a displayable result. To do this:

- for each element, the macro \tplformat, requiring 2 mandatory arguments, is executed. The first argument passed is the index of the element and the 2nd argument is the element itself;
- each result from the macro \tplformat is separated from the next by the content of the macro \tplsep.

By default, these two macros have the following code:

<sup>&</sup>lt;sup>5</sup>If p is the argument of the method, we define h = (n-1)p+1 where n is the length of the tuple. The method returns the number equal to  $x_{\lfloor h \rfloor} + (h-\lfloor h \rfloor)(x_{\lfloor h \rfloor} + x_{\lceil h \rceil})$ , where  $x_k$  is the  $k^{\text{th}}$  number of the sorted tuple.

```
\def\tplformat#1#2{#2}% #1=current index #2=current item
\def\tplsep{, }
```

The default behavior is therefore exactly the same as the implicit method that is executed last, and in particular, the method is expandable.

These 2 macros can be reprogrammed to create more advanced formatting. Here we use the \tplfpcompare macro, which is an alias of the \fp\_compare:nNnTF macro of the l3fp module, to compare an element with a given value. In the 2 examples given below, the method is no longer expandable due to the use of the \fbox and \textcolor macros.

Boxing the first 10 elements:

Below-average items highlighted in red:

## 5 Methods of the group tuple $\longrightarrow$ tuple

Each time a tuple is generated or modified, the len, sum, min, max, mean, med, stdev and sorted methods are updated.

#### 5.1 Method sorted

This expandable method does not accept any arguments and returns a tuple object with its elements sorted in ascending order.

#### 5.2 Method set

The syntax of this expandable method is

```
\mathtt{set}\{\langle \mathit{index1}\rangle : \langle \mathit{number1}\rangle, \langle \mathit{index2}\rangle : \langle \mathit{number2}\rangle, \ldots\}
```

In the tuple resulting from the previous methods, replaces the number at  $\langle index1 \rangle$  with  $\langle number1 \rangle$  and so on if several assignments are specified in a comma-separated list.

Each  $\langle index \rangle$  must be between 0 and n-1, where n is the number of elements in the tuple passed as input to the method. Negative indexes from -n to -1 are also permitted.

#### 5.3 Method add

This expandable method, which adds a (insertion) at one or more specified indexes, has the syntax

```
add\{\langle index1\rangle:\langle insertion1\rangle,\langle index2\rangle:\langle insertion2\rangle,...\}
```

It should be noted that in this syntax, a (insertion) can be

- a single number "add{\langle index\rangle : \langle number\rangle \rangle "
- a csv list of numbers that *must* be enclosed in braces "add{\(\langle index\):{\(n1,n2,n3...\)}\)"
- a tuple accessed by \tplexe: "add{\langle index\rangle: \tplexe{\langle tuple name\rangle}}\rangle\rangle."

As for  $\langle index \rangle$ , they can be between 0 and n, where n is the number of elements in the tuple. Negative indexes between -n-1 and -1 are also permitted:

- a  $\langle index \rangle$  equal to 0 or -n-1 places the  $\langle insertion \rangle$  at the beginning of the tuple passed as input;
- a  $\langle index \rangle$  equal to n or -1 places the  $\langle insertion \rangle$  at the end of the tuple;
- the \(\langle index \rangle \) are not updated after each \(\langle insertion \rangle \), but only after the last one. It is therefore not equivalent to write ".add{1:100,2:200}" and ".add{1:100}.add{2:200}". Indeed, 200 will be at index 3 in the first case, whereas it will be at index 2 in the second.

```
\newtuple{X}{10,20,30,40,50,60}%
\newtuple{Y}{1,2,3,4}%

1) \tplexe{X.add{0:\{\tplexe{Y}\}}\par

2) 10, 20, 1, 2, 3, 4, 30, 40, 50, 60

2) \tplexe{X.add{2:\{\tplexe{Y}\}}\par

3) \tplexe{X.add{-1:\{\tplexe{Y}\}}\par

4) \tplexe{X.add{1:100,2:200}\par

5) \tplexe{X.add{1:100}.add{2:200}}
```

## 5.4 Method op

This expandable method, which performs an *(operation)* on all elements of the tuple, has the syntax

```
op{\langle operation \rangle \}
```

The *(operation)* is an expression not containing braces, evaluable by \fpeval once all occurrences of "val" have been replaced by the value of each element, and all occurrences of "idx" by its index.

```
\newtuple{X}{10,20,30,40,50,60}%

1) \tplexe{X.op{val+5}}\par

2) \tplexe{X.op{val*val}}\par

3) \tplexe{X.op{val+idx}}\par

4) \tplexe{X.op{idx<4 ? val-1 : val+1 }}

1) 15, 25, 35, 45, 55, 65

2) 100, 400, 900, 1600, 2500, 3600

3) 10, 21, 32, 43, 54, 65

4) 9, 19, 29, 39, 51, 61
```

## 5.5 Method filter

Thi expandables method, which selects elements according to one or more criteria, has the syntax

```
\texttt{filter}\{\langle \textit{test}\rangle\}
```

and where  $\langle test \rangle$  is a boolean not containing braces, evaluable by \fpeval once all occurrences of "val" have been replaced by the value of each element, and all occurrences of "idx" by its index.

## 5.6 Method comp

This expandable method composes two tuples of the same length with an operation that the user specifies. Its syntax is

```
comp\{\langle operation \rangle\}\{\langle tuple name \rangle\}
```

where the tuple whose name is passed as the second argument *must* have the same length as the tuple passed as input to the method.

The *(operation)* is an expression not containing braces, evaluable by \fpeval once all occurrences of "xa" have been replaced by the value of each element of the input tuple, and all occurrences of "xb" by that of the tuple specified in the 2nd argument.

Product of 2 tuples and their "sumprod":

```
\newtuple{A}{2,-4,3,7,-1}%
\newtuple{B}{-9,0,4,6,-2}% product \tplexe{A.comp{xa*xb}{B}}\par sumprod: \tplexe{A.comp{xa*xb}{B}.sum}

product -18, -0, 12, 42, 2 sumprod: 38
```

Calculation of the smallest distance to point A(2.5; -0.5) knowing the list of abscissas and the list of ordinates of a trajectory (here elliptical):

```
\newtuple{ListX}{4,2,0.5,1,3,6.5}%
\newtuple{ListY}{2,1.5,0,-1.5,-2,0.5}%
\tplexe{ListX.comp{sqrt((xa-2.5)**2+(xb+0.5)**2)}{ListY}.min}
```

## 6 Methods in the group tuple → storage

As these methods don't return a result because they perform an assignment, they are not expandable, and must be placed in the last position. If this is not the case, all methods following them will be ignored.

## 6.1 Method split

This method cuts the tuple passed as input to the method after the specified index. The syntax is

```
split{\langle index \rangle}{\langle tuple1 \rangle}{\langle tuple2 \rangle}
```

The tuple passed as input to the method is split after the  $\langle index \rangle$ : the part before the split is assigned, via newtuple to the tuple with name "tuple1" and the remaining part to the tuple with name "tuple2". No check is made on the existence of the 2 tuples, so existing tuples can be silently replaced.

The  $\langle index \rangle$  must lie between 0 and n-2 if positive, or between -n and -2 if negative, n being the number of elements in the tuple passed as input.

```
%\newtuple{nn}{12.7, 6.3, 11.7, 2.9, 5.5, 8.1, 4.3, 9.4,

% 13.6, 2.9, 6.9, 11.2, 5.1, 7.7, 10.1, 3, 4.1}%

\tplexe{nn.split{5}{n1}{n2}}%

tuple before: \tplexe{n1}\par

tuple after: \tplexe{n2}
```

#### 6.2 Method store

This macro is used to store the result of the last method. If this result is a tuple, the syntax is

```
store{⟨tuple name⟩}
```

and if the result is a number or a displayable data from the show method:

```
store{\langle macro\range}
```

No check is made on the existence of the tuple or macro. It is therefore possible to silently replace an existing tuple or macro.

## 7 Tuple generation

To generate a tuple, we can use the expandable macro \gentuple which is intended to be called in the 2nd argument of \newtuple. Its syntax is

#### where:

- the ⟨initial values⟩ are optional. If present, they must be followed by a comma. The macro \gentuple determines their number i by counting (i must be at most equal to 9). These initial values will be copied at the beginning of the tuple and subsequently, are intended to be used in the ⟨generation rule⟩ for recurrence purposes;
- the \(\(\lambda\) generation \(rule\)\) is an expression not containing braces, evaluable by \fpeval once in the previous \(i\) values, all occurrences of \1 have been replaced by the first value, \2 by the second value, etc. In addition, each occurrence of \i is replaced by the value of the current index.
- the \(\lambda condition\rangle\) is a boolean not containing braces, evaluable by \fpeval once all occurrences of "val" have been replaced by the value of the computed element, and all occurrences of "\i" by its index. If the keyword after; is \while, the loop is of the type while \(\lambda condition\rangle \ldots endwhile\) whereas if this keyword is \until, it is a repeat...until\(\lambda condition\rangle\) loop.

Generating the first 10 even integers:

\ge	entuple{\genrule (\i+1)*2; \until \i=9 }\par	2, 4, 6, 8, 10, 12, 14, 16, 18, 20
or'	ypar	or
\ge	entuple{\genrule (\i+1)*2; \while \i<10}	2, 4, 6, 8, 10, 12, 14, 16, 18, 20

Generation of 15 random integers between 1 and 10:

\gentuple{\genrule randint(1,10) ; \until \i=14 } 5, 3, 2, 5, 4, 7, 6, 4, 3,	5, 5, 8, 7, 1, 3
--	------------------

Generate squares of integers up to 500:

```
\gentuple{\genrule\i*\i; \while val<500 } 0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400, 441, 484
```

Generation of the first 10 terms of the Fibonacci sequence:

```
\gentuple{1,1,\genrule\1+\2; \until \i=9 } 1,1,2,3,5,8,13,21,34,55
```

Generation of the first 10 terms of  $u_0 = 1$ ;  $u_1 = 1$ ;  $u_2 = -1$  and  $u_n = u_{n-3}u_{n-1} - u_{n-2}^2$ :

```
\gentuple{1,1,-1,\genrule\1*\3-\2*\2; \until \i=10} 1,1,-1,-2,-3,-1,-7,20,-69,83,-3101
```

Generation of the Syracuse sequence (aka the "3n + 1 sequence") of 15:

```
\gentuple{15,\genrule \1/2=trunc(\1/2) ? \1/2 : 3*\1+1 ; \until val=1}% 15,46, 23, 70, 35, 106, 53, 160, 80, 40, 20, 10, 5, 16, 8, 4, 2, \text{undefined}
```

Maximum altitude and length of the Syracuse sequence of 27:

```
\newtuple{syr27} {\gentuple{27,\genrule \1/2=trunc(\1/2) ? \1/2 : 3*\1+1 ; \until val=1}}% length = 112 length = \tplexe{syr27.len}\par max alt = \tplexe{syr27.max} max alt = \tplexe{syr27.max}
```

## 8 Conclusion

#### 8.1 Motivation

Somewhat surprisingly, very little exists to manipulate and operate on lists of numbers<sup>6</sup>.

The main challenge was to provide expandable macros and also an original syntax in the world of  $T_EX$  of the type  $\langle object \rangle$ .  $\langle method\ 1 \rangle$ .  $\langle method\ 2 \rangle$ ... $\langle method\ n \rangle$ , where you can chain methods executed on an "object" which is a tuple of numbers. I don't know if any packages offer this kind of syntax, but it's actually quite intuitive. As I'm fairly unfamiliar with programming expandable macros, I almost gave up many times. But I've finally found a mouse-hole that makes it all work pretty much, except for the numerous bugs that are probably lurking everywhere.

I'd like to thank anyone who finds one for pointing it out to me by e-mail, or even suggesting new features.

## 8.2 Execution speed

When you get into optimizing execution speed, especially for expandable macros, you don't get out! It's a pit of questions about macro arguments, about little — or big — tricks that save time, and a headache about how to juggle with delimited arguments and find them later!

I hope I didn't fall into this trap, because I've barely entered it. In any case, TEX is not made for massive calculations, since it is first and foremost a typesetting software. For calculations, powerful tools that beat TEX hands down exist in abundance.

In any case, "expandable macro" goes a bit against "execution speed". For information, I put below the compilation times, in seconds, of the creation of tuples containing n random integers. It depends on the computer used, of course, but the orders of magnitude are quite revealing. We can clearly see that it is illusory to exceed a thousand numbers because the time to create a tuple then increases rapidly<sup>7</sup>. That said, who would use  $T_EX$  for calculations on so many numbers?

This table illustrates the creation speeds of a tuple of random signed integers between -1000 and 1000. As a reminder, creating a tuple involves sorting its elements (by quick sort), calculating the sum of the numbers and the sum of squares. The 3 modes can thus be compared for larger or smaller tuples.

Nb items	int	dec:short	dec:long
25	0.001 s	0.001 s	0.006 s
50	0.002 s	0.004 s	0.016 s
100	0.005 s	0.008 s	0.031 s
200	0.019 s	0.027 s	0.082 s
400	0.082 s	0.103 s	0.227 s
800	0.427 s	0.513 s	0.769 s
1600	3.349 s	3.939 s	4.355 s

<sup>&</sup>lt;sup>6</sup>There is one package, commalists-tools, but it's clearly too limited. Like all its author's packages that flood CTAN, it simply aligns, linearly and without any real programmation, the high-level macros of the packages tikz, listofitems, xstring, xint and simplekv.

<sup>&</sup>lt;sup>7</sup>Not to mention that, in addition to that, the tuple is immediately recreated and recalculated after being modified by the methods set, add, op, filter, split and comp

## 8.3 Example: state population

The tuple \Wpop contains the population of each state in the world, in millions of inhabitants<sup>8</sup>:

```
43.4, 2.8, 46.3, 37.8, 0.1, 46.1, 2.8, 0.1, 26.7, 9.0, 10.5, 0.4, 1.5,
174.7, 0.3, 9.5, 11.7, 0.4, 14.1, 0.8, 12.6, 3.2, 2.7, 217.6, 0.5, 6.6,
23.8, 13.6, 0.6, 17.1, 29.4, 39.1, 5.9, 18.8, 19.7, 1425.2, 7.5, 0.7,
52.3, 0.9, 6.2, 5.2, 29.6, 4.0, 11.2, 0.2, 1.3, 10.5, 26.2, 105.6, 5.9,
1.2, 0.1, 11.4, 18.4, 114.5, 6.4, 1.8, 3.8, 1.3, 1.2, 129.7, 0.9, 5.5,
64.9, 0.3, 0.3, 2.5, 2.8, 3.7, 83.3, 34.8, 10.3, 0.1, 0.4, 0.2, 18.4,
14.5, 2.2, 0.8, 11.9, 10.8, 10.0, 0.4, 1441.7, 279.8, 89.8, 46.5, 5.1,
9.3, 58.7, 2.8, 122.6, 11.4, 19.8, 56.2, 0.1, 4.3, 6.8, 7.7, 1.8, 5.2,
                                                                             Number of state: 204
2.4, 5.5, 7.0, 2.7, 0.7, 31.1, 21.5, 34.7, 0.5, 24.0, 0.5, 0.4, 5.0,
                                                                            Mean: 39.66338235294118
1.3 ,129.4, 0.1, 3.5, 0.6, 38.2, 34.9, 55.0, 2.6, 31.2, 17.7, 0.3, 5.3,
                                                                            Median: 7.6
7.1 ,28.2, 229.2, 2.1, 5.5, 4.7, 245.2, 4.5, 10.5, 6.9, 34.7, 119.1,
                                                                            Standard deviation: 146.8985787142461
40.2 ,10.2, 3.3, 2.7, 51.7, 3.3, 1.0, 19.6, 144.0, 14.4, 0.0, 0.2, 0.1,
                                                                            Quintile #1: 0.8
0.2,0.03, 0.2, 37.5, 18.2, 7.1, 0.1, 9.0, 6.1, 0.0, 5.7, 2.1, 0.8, 18.7,
                                                                            Quintile #4: 37.62
61.0 ,11.3, 47.5, 21.9, 5.5, 49.4, 0.6, 10.7, 8.9, 24.3, 10.3, 71.9, 1.4,
9.3, 0.1, 1.5, 12.6, 86.3, 6.6, 0.0, 0.0, 49.9, 37.9, 9.6, 68.0, 69.4,
341.8 ,0.1, 3.4, 35.7, 0.3, 29.4, 99.5, 0.6, 35.2, 21.1, 17.0}
Number of state: \tplexe{\Wpop.len}\par
Mean: \tplexe{\Wpop.mean}\par
Median: \tplexe{\Wpop.med}\par
Standard deviation: \tplexe{\Wpop.stdev}\par
Quintile \#1 : \tplexe{\Wpop.quantile{0.2}}\par
Quintile \#4 : \tplexe{\Wpop.quantile{0.8}}
```

We modify the tuple \Wpop, retaining only "moderately" populated states. We arbitrarily consider their population between 10 and 100 millions:

```
\tplexe{\Wpop.filter{val>=10 && val<=100}.store\Wpop}%
Number: \tplexe{\Wpop.len}\par
                                                                              Number: 79
Mean: \tplexe{\Wpop.mean}\par
                                                                              Mean: 32.33037974683544
Median: \tplexe{\Wpop.med}\par
                                                                              Median: 26.7
Standard deviation: \tplexe{\Wpop.stdev}\par
                                                                              Standard deviation: 21.22899470798109
Quintile \#1 : \tplexe{\Wpop.quantile{0.2}}\par
                                                                              Ouintile #1: 12.6
Quintile \#4 : \tplexe{\Wpop.quantile{0.8}}
                                                                              Quintile #4: 48.26
                                                                              Distribution over 6 equal intervals:
Distribution over 6 equal intervals:\par
\begin{tabular}{lc}\\hline
                                                                                From 10 to 25
                                                                                                38
    From 10 to 25 & \t \ \tplexe{\Wpop.filter{val<25}.len}\\
                                                                                From 25 to 40
                                                                                                19
    From 25 to 40 & \t {\mathbb Wpop.filter{val}=25 \& val<40}.len}
                                                                                From 40 to 55
                                                                                                10
    From 40 to 55 & \t \ \text{\Wpop.filter{val>=40 && val<55}.len}\\
                                                                                From 55 to 70
                                                                                                7
    From 55 to 70 & \t {\mathbb Wpop.filter}  & val<70}.len}\\
                                                                                From 70 to 85
                                                                                                2
    From 70 to 85 & \tplexe{\Wpop.filter{val>=70 && val<85}.len}\\</pre>
                                                                                From 85 to 100
    From 85 to 100& \t \000. filter{val>=85}.len}\\\
\end{tabular}
```

## 8.4 TODO list

To be implemented more or less quickly:

- 1. insertion sorting to sort almost-sorted tuples obtained using the add, set, op methods (risky as it depends on the operation!).
- 2. merge sorting to add one tuple to another;
- 3. other speed optimization?

<sup>&</sup>lt;sup>8</sup>The data comes from https://www.unfpa.org/data/world-population-dashboard